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MP MCA Entrance Test
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We are happy to present a comprehensive analysis of this year's MP MCA entrance test. It has been recreated with the help of PT faculty and PT students from across the nation.

The entrance test for MP MCA had no change in its pattern as compared to the last year. There were 100 questions to be solved in 120 minutes. There was **no Hindi version** and **no negative marking.** The pattern of the questions was easy. Only difference was the weightage of Logical Reasoning portion increased compared to the last year.

A bird's eye view :

Total Number of Questions	:	100
Total Time	:	120 minutes
Marking pattern	:	No Negative marking.

The following is the number of questions against each topic :

Coordinate Geometry	:	17
Differential Calculus	:	9
Integral Calculus	:	13
Algebra	:	17
Statistics	:	18
Numerical Analysis	:	4
Computers	:	10
Logical Reasoning	:	12

Disclaimer: All these questions have been memorised by PT students. We are merely reproducing a few of them here in fragments to ensure that the huge community of students eagerly waiting to see an objective comparison of their performance gets the right picture.

DIRECTIONS : *For the following questions choose the correct option.*

1. If f(x) = log x, then f'(x) at x = log 4 is :

(1) $-\log 4$ (2) $\log \frac{1}{4}$ (3) $\frac{1}{\log 4}$ (4) none of these.

Sol. $f'(x) = \frac{1}{x}$, at x = log 4, $f'(x) = \frac{1}{\log 4}$. **Ans.(3)**

2. $\int_0^1 e^{\sqrt{x}}\,dx =$

(1) 2e – 2 (2) 2e (3) 4e (4) 4e – 2

Sol. Let $\sqrt{x} = t$, so $I = 2\int_0^1 te^t\,dt = 2\left[(t+1)e^t\right]_0^1$

$= 2[2e - 1] = 4e - 2$. **Ans.(4)**

3. Probability of drawing a king or a spade from a pack of well shuffled cards is :

(1) $\frac{17}{52}$ (2) $\frac{4}{13}$ (3) $\frac{2}{13}$ (4) none of these

Sol. P(king or spade) = $\frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$. **Ans.(2)**

4. The probability of P, Q, R passing an examination are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ respectively. Then the probability that exactly one of them passes the examination is :

(1) $\frac{1}{24}$ (2) $\frac{11}{24}$ (3) $\frac{23}{24}$ (4) $\frac{13}{24}$

Sol. Required probability = $P(P\,\overline{Q}\,\overline{R}) + P(\overline{P}\,Q\,\overline{R}) + P(\overline{P}\,\overline{Q}\,R)$

$= \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} + \frac{1}{2} \times \frac{1}{3} \times \frac{3}{4} + \frac{1}{2} \times \frac{2}{3} \times \frac{1}{4} = \frac{11}{24}$. **Ans.(2)**

5. A man and his wife appear for an interview for two posts. The probability of husband's selection is $\frac{1}{7}$ and wife's is $\frac{1}{5}$. What is the probability that both will be selected ?

(1) $\frac{1}{35}$ (2) $\frac{34}{35}$ (3) $\frac{12}{35}$ (4) $\frac{2}{35}$

Sol. $P(H \cap W) = \frac{1}{7} \cdot \frac{1}{5} = \frac{1}{35}$. **Ans.(1)**

6. Telephone calls in a exchange follows Poisson distribution. The average number of calls in a day is 2.5. Then find the probability that on a particular day no call comes into the exchange.

(1) 0.0821 (2) 0.0135 (3) 2.5 (4) 6.25

Sol. P(no call) = $e^{-2.5}$ = 0.0821. **Ans.(1)**

7. In a normal distribution the standard deviation is 25 and mean is 5, then P(20 < x < 30) is equal to:

(1) 0.3413 (2) 0.6826 (3) 0.1587 (4) 0.5

Sol. Transforming the normal variate X to standard normal variate z by the equation

$$z = \frac{X - M}{\sigma}$$

For x = 20 and X = 30, the corresponding value of z will be – 1 and 1.

$P(-1 \le z \le 1) = 2z(1) = 2(0.3413) = 0.6826$. **Ans.(2)**

8. If the random variable X has Binomial distribution with mean 4 and variance 3, then the parameters n, p are :

(1) 16, 1/4 (2) 12, 1/4 (3) 16, 3/4 (4) none of these

Sol. np = 4, npq = 3 $\Rightarrow q = \frac{3}{4}, p = \frac{1}{4}, n = 16$. **Ans.(1)**

9. $\frac{n!}{r!\,(n-r)!} =$

(1) ${}^nC_r$ (2) ${}^nP_r$ (3) ${}^nC_r r!$ (4) none of these

Sol. **Ans.(1)**

10. The middle term in the expansion of $\left(1 - \frac{x^2}{2}\right)^{14}$ is :

(1) $\left(1 - \frac{x^2}{2}\right)^{14}$ (2) $-\frac{429}{16}x^{14}$ (3) $\frac{716}{16}x^{14}$ (4) none of these

Sol. Middle term of $\left(1 - \frac{x^2}{2}\right)^{14}$ is $= {}^{14}C_7 (1)^7 \left(-\frac{x^2}{2}\right)^7 = -\frac{429}{16}x^{14}$. **Ans. (2)**

11. In $(5 - 4x)^{-7}$ when $x = \frac{1}{2}$, greatest term is :

(1) $t_4 = t_5$ (2) $t_3 = t_4$ (3) t_2 (4) $t_2 = t_3$

Sol. Let T_{r+1} be the greatest term in $(5 - 4x)^{-7}$,

when $x = \frac{1}{2}$, then $\frac{T_{r+1}}{T_r} = \frac{n - r + 1}{r}\left(\frac{a}{x}\right) = \frac{-7 - r + 1}{r}\left(\frac{-4x}{5}\right) > 1$

or $\frac{(6 + r)}{r}\frac{4}{5}\cdot\frac{1}{2} > 1$ or 24 + 4r > 10r or 24 > 6r or 4 > r.

Since 4 is an integer, 4[th] and 5[th] term are greatest. **Ans.(1)**

12. Coefficient of x^{10} in the expansion of $\left(2x^3 - \frac{1}{x}\right)^{10}$ is :

(1) – 4032 (2) 4032 (3) – 8064 (4) none of these

Sol. Let $(m + 1)^{th}$ term contains x^{10}, where $m = \frac{10(3) - 10}{4} = 5$

$\therefore T_6 = T_{5+1} = {}^{10}C_5\, 2^5\, (-1)^5 = -8064$. **Ans.(3)**

13. If $r.{}^nC_r = 13\ {}^{n-1}C_{r-1}$, then n is equal to :

(1) 13 (2) 26 (3) 78 (4) none of these

Sol. $r\ {}^nC_r = 13\ {}^{n-1}C_{r-1} \Rightarrow r \cdot \frac{n}{r}\ {}^{n-1}C_{r-1} = 13\ {}^{n-1}C_{r-1} \Rightarrow n = 13$. **Ans.(1)**

14. $\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} \ldots$ is equal to :

(1) e (2) e^{-1} (3) 2e (4) none of these

Sol. $e^{-1} = 1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \ldots = \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \ldots$. **Ans.(2)**

15. $x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \ldots =$

(1) log x (2) $\frac{e^x - e^x}{2}$ (3) $\frac{1}{2}\log\left(\frac{1+x}{1-x}\right)$ (4) $\frac{1}{2}\log\left(\frac{1-x}{1+x}\right)$

Sol. Standard result. **Ans.(3)**

16. If $y = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots$ and if $|x| < 1$, then x is equal to :

(1) e^y (2) $e^y - 1$ (3) log (1 + y) (4) log (1 – y)

Sol. $y = \log_e (1 + x)$

$\Rightarrow e^y = 1 + x \Rightarrow x = e^y - 1$. **Ans.(2)**

17. If $A_\alpha = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, then $A_{\alpha+\beta}$ equals :

(1) $A_\alpha + A_\beta$ (2) $A_\alpha - A_\beta$ (3) $A_\alpha . A_\beta$ (4) none of these

Sol. $A_{\alpha+\beta} = \begin{bmatrix} \cos(\alpha+\beta) & \sin(\alpha+\beta) \\ -\sin(\alpha+\beta) & \cos(\alpha+\beta) \end{bmatrix}$

$$= \begin{bmatrix} \cos\alpha\cos\beta - \sin\alpha\sin\beta & \sin\alpha\cos\beta + \cos\alpha\sin\beta \\ -(\sin\alpha\cos\beta + \cos\alpha\sin\beta) & \cos\alpha\cos\beta - \sin\alpha\sin\beta \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \cdot \begin{bmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{bmatrix} = A_\alpha . A_\beta$$ **Ans.(3)**

18. The rank of the matrix $A = \begin{bmatrix} 1 & -3 & 2 \\ 3 & -9 & 6 \\ -2 & 6 & -4 \end{bmatrix}$ is :

(1) 0 (2) 1 (3) 2 (4) 3

Sol. $|A| = \begin{vmatrix} 1 & -3 & 2 \\ 3 & -9 & 6 \\ -2 & 6 & -4 \end{vmatrix} = 0$ [$\because$ The columns are proportional]

Therefore $\rho(A) < 3$. Also there exist no minor of order 2 of A which is not equal to zero. $\rho(A) < 2$ .....(i). Finally, since all the minor of order one of the matrix A are not zero. Therefore $\rho(A) = 1$ .....(ii). **Ans.(2)**

19. The system of equations $x + y + z = k_1$, $2x + 3y - 2z = k_2$, $5x + y + 2z = k_3$ has :
(1) infinite solutions (2) unique solution (3) two solution (4) no solution

Sol. Consider the system of equations $a_1x + b_1y + c_1z = k_1$, $a_2x + b_2y + c_2z = k_2$, $a_3x + b_3y + c_3z = k_3$. If the determinant of coefficient matrix is zero (i.e. matrix is non-singular) then, the system of equations possesses unique solution. Determinant of coefficient matrix

$$= \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & -2 \\ 5 & 1 & 2 \end{vmatrix} = 1 \times (6 + 2) - 1(4 + 10) + 1(2 - 15) = 8 - 14 - 13 = -19.$$

That is the matrix is non-singular and hence, the system has unique solution. **Ans.(2)**

20. $\begin{bmatrix} 0 & -3 & 4 \\ 3 & 0 & 2 \\ -4 & -2 & 0 \end{bmatrix}$ is a :

(1) Symmetric matrix (2) skew symmetric matrix
(3) idempotent matrix (4) none of these.

Sol. Ans.(2)

21. Inverse of $\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$ is :

(1) $\frac{1}{25}\begin{bmatrix} 25 & -10 & -15 \\ -10 & 4 & 11 \\ -15 & 1 & 9 \end{bmatrix}$ (2) $\frac{1}{25}\begin{bmatrix} 5 & -15 & -25 \\ -10 & 2 & 13 \\ -15 & 1 & 9 \end{bmatrix}$

(3) $\begin{bmatrix} 25 & -10 & -15 \\ 15 & 1 & 9 \\ 10 & 4 & -11 \end{bmatrix}$ (4) none of these

Sol. $\begin{vmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{vmatrix} = 1(25) + 3(-10) - 2(-15) = 25$, adj $A = \begin{bmatrix} 25 & -10 & -15 \\ -10 & 4 & +11 \\ -15 & 1 & 9 \end{bmatrix}$.

$\therefore A^{-1} = \frac{1}{25}\begin{bmatrix} 25 & -10 & -15 \\ -10 & 4 & 11 \\ -15 & 1 & 9 \end{bmatrix}$. **Ans.(1)**

22. Minor of a_{23} for the matrix $\begin{bmatrix} 4 & 5 & 6 \\ -3 & 2 & 1 \\ 8 & 6 & 3 \end{bmatrix}$ is :

(1) – 16 (2) 16 (3) 22 (4) – 22

Sol. $M_{23} = \begin{vmatrix} 4 & 5 \\ 8 & 6 \end{vmatrix} = -16$. **Ans.(1)**

23. Cofactor of element – 4 for the matrix $\begin{vmatrix} 1 & 2 & 3 \\ -3 & 2 & -1 \\ 2 & -4 & 3 \end{vmatrix}$ is :

(1) 8 (2) – 8 (3) 10 (4) – 10

Sol. $C_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 3 \\ -3 & -1 \end{vmatrix} = -8$. **Ans.(2)**

24. If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$, then A^{-1} is equal to :

(1) $\frac{1}{2}(A^2 + 6A - 7I_3)$ (2) $\frac{1}{2}(A^2 - 6A - 7I_3)$

(3) $-\frac{1}{2}(A^2 - 6A + 7I_3)$ (4) $\frac{1}{2}(2A^2 - 5A + 7I_3)$

Sol. The characteristic equation of A is $|A - \lambda I_3| = 0$

$\Rightarrow \begin{vmatrix} 1-\lambda & 0 & 2 \\ 0 & 2-\lambda & 1 \\ 2 & 0 & 3-\lambda \end{vmatrix} = 0 \Rightarrow \lambda^3 - 6\lambda^2 + 7\lambda + 2 = 0.$

$\therefore A^3 - 6A^2 + 7A + 2I_3 = 0$

$\Rightarrow A^2 - 6A + 7I_3 + 2A^{-1} = 0 \Rightarrow A^{-1} = -1/2(A^2 - 6A + 7I_3)$. **Ans.(3)**

25. $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & -2 \end{bmatrix} + \begin{bmatrix} 1 & 4 & 5 \\ 6 & -3 & 8 \\ 4 & -2 & 6 \end{bmatrix} = \begin{bmatrix} x & 3 & 6 \\ 5 & y & 7 \\ 5 & -3 & z \end{bmatrix}$ then x, y, z are respectively equal to :

(1) 3, – 5, 8 (2) 3, – 1, – 4 (3) 3, 1, 4 (4) none of these

Sol. x = 3, y = – 1, y = 4. **Ans.(4)**

26. $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} =$

(1) $(a - b)(b - c)(c - a)$ (2) $(a + b + c)(ab + bc + ca)$
(3) $3abc - (a^3 + b^3 + c^3)$ (4) none of these

Sol. $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = a(bc - a^2) - b(b^2 - ac) + c(ba - c^2) = 3abc - (a^3 + b^3 + c^3)$. **Ans.(3)**

27. Name the register which contains the address of next instruction ?
(1) Program counter (2) Memory address register
(3) Memory Buffer register (4) none of these.

Sol. Ans.(1)

28. Name the register which contain the address of stack ?
(1) stack pointer (2) Program counter
(3) Program status word (4) none of these.

Sol. Ans.(1)

29. Set of instructions which can be used many time in a program is :
(1) array (2) pointer (3) subroutine (4) loop

Sol. Ans.(3)

30. ENIAC was manufactured by :
(1) Charles Babbage (2) Herman Hollerith (3) Eckert and Mauchy (4) none of these

Sol. Ans.(3)

31. The decimal number $(132.8125)_{10}$ when converted to binary system gives :
(1) $(10000100.1101)_2$ (2) $(11000100.1011)_2$
(3) $(11000101.1101)_2$ (4) $(10000100.1011)_2$

Sol. The integral part 132 can be converted to binary system as follows :

2	132	Rem
2	66	– 0
2	33	– 0
2	16	– 1
2	8	– 0
2	4	– 0
2	2	– 0
	1	– 0

(132)10 = (10000100)2

The fractional part 0.8125 can be converted to binary system as follows :

	Integer Part
$0.8125 \times 2 = 1.6250$	1
$0.6250 \times 2 = 1.2500$	1
$0.2500 \times 2 = 0.5000$	0
$0.5000 \times 2 = 1.000$	1

So $(0.8125)_{10} = (0.1101)_2$.
Hence $(132.8125)_{10} = (10000100.1101)_2$. **Ans.(1)**

32. $11001_2 - 10011_2$ is equal to :

(1) 110_2 (2) 111_2 (3) 101_2 (4) 1110_2

Sol.
$$\begin{array}{r} 11001 \\ -\,10011 \\ \hline 00110 \end{array}$$
. **Ans.(1)**

33. The decimal equivalent of hexadecimal number 4B3 is :

(1) 1200 (2) 1230 (3) 1203 (4) 1215

Sol. $4B3 = 4 \times 16^2 + 11 \times 16 + 3 \times 1 = (1203)_{10}$. **Ans.(3)**

34. Binary 1001 multiplied by binary 1000 is equivalent to :

(1) 1001001 (2) 1001000 (3) 10001 (4) 10101011

Sol. $1001 \times 1000 = 1001000$. **Ans.(2)**

35. How many bits are there in a byte ?

(1) 84 (2) 8 (3) 16 (4) 32

Sol. **Ans.(2)**

36. If x = 8, y = 9, z = 6, then the expression z ↑ 3 + 3 * z/y – 4*x*z is equal to :

(1) 27.66 (2) 26.33 (3) 26 (4) none of these

Sol. z ↑ 3 + 3 * z/y – 4 * x * z = $(6)^3 + 3(6)/9 - 4(8)(6) = 26$. **Ans.(3)**

37. Eccentricity of circle is :

(1) 0 (2) 1 (3) not defined (4) none of these.

Sol. **Ans.(1)**

38. Eccentricity of rectangular hyperbola is :

(1) 1 (2) $\sqrt{2}$ (3) $1/\sqrt{2}$ (4) none of these.

Sol. **Ans.(2)**

39. Line $y = mx + c$ will be tangent to the circle $x^2 + y^2 = a^2$ if :

(1) $c^2 = a^2(1 + m^2)$ (2) $c^2 = a(1 + m^2)$ (3) $c^2 = a(1 - m^2)$ (4) $c^2 = a^2 + m^2$

Sol. Standard result. **Ans.(1)**

40. The equation to the circle which passes through the origin and cuts orthogonally each of the circles $x^2 + y^2 - 6x + 8 = 0$ and $x^2 + y^2 - 2x - 2y = 7$ is :

(1) $3x^2 + 3y^2 + 8x + 29y = 0$ (2) $3x^2 + 3y^2 - 8x + 29y = 0$

(3) $3x^2 + 3y^2 - 8x - 29y = 0$ (4) none of these

Sol. Any circle passing through the origin may be given by

$x^2 + y^2 + 2gx + 2fy = 0$ ...(1)

The two circles are given as $x^2 + y^2 - 6x + 8 = 0$ ...(2)

and $x^2 + y^2 - 2x - 2y - 7 = 0$. ...(3).

If (1) and (2) cut orthogonally, then $2g(-3) + 2.f.0 = 0 + 8$, $g = -4/3$. ...(4).

If (1) and (3) cut orthogonally, then $2g(-1) + 2f(-1) = 0 - 7$

Putting the value of g from (4) and solving, $f = 29/6$.

Substituting in (3), we get $3x^2 + 3y^2 - 8x + 29y = 0$. **Ans.(2)**

41. The pole of the line $8x - 2y = 11$, with respect t the circle $2x^2 + 2y^2 = 11$ is :
(1) (4, – 1) (2) (4, 1) (3) (–4, 1) (4) none of these

Sol. Given circle is $x^2 + y^2 = \frac{11}{2}$ and the line is $8x - 2y - 11 = 0$

$\therefore$ Pole $= \left(\frac{-lr^2}{n}, \frac{-mr^2}{n}\right) = \left(\frac{-8(11/2)}{-11}, \frac{-(-2)(11/2)}{-11}\right) = (4, -1)$. **Ans.(1)**

42. If $x^2 + y^2 - 2x = 0$ is a member of co-axial system for which (4, 4) is limiting point, then radical axis of the system is :
(1) $6x + 8y - 32 = 0$ (2) $6x - 8y + 32 = 0$ (3) $6x - 8y - 32 = 0$ (4) none of these

Sol. The given circle is $x^2 + y^2 - 2x = 0$ ... (i)
The equation of the circle, for which (4, 4) is the limiting point is given by
$x^2 - 8x + y^2 - 8y + 32 = 0$ ... (ii)
Radical axis of (i) and (ii) is $6x + 8y - 32 = 0$. **Ans.(1)**

43. The angle of intersection of circles $x^2 + y^2 = 4$ and $x^2 + y^2 - 2x - 2y = 0$ is :
(1) 30° (2) 45° (3) 60° (4) 90°

Sol. Here the radius of the given circles $x^2 + y^2 = 4$ and $x^2 + y^2 - 2x - 2y = 0$ are 2 and $\sqrt{2}$ respectively. and their centres are (0, 0) and (1, 1) respectively.

$\cos\theta = \frac{r_1^2 + r_2^2 - d^2}{2r_1r_2} = \frac{1}{\sqrt{2}} \Rightarrow \theta = 45°$. **Ans.(2)**

44. The length of tangent drawn to the circle $x^2 + y^2 - 2x - 10y + 1 = 0$ from (– 2, – 3) is :
(1) 48 (2) $4\sqrt{3}$ (3) $16\sqrt{3}$ (4) none of these

Sol. Length of tangent $= \sqrt{(-2)^2 + (-3)^2 - 2(-2) - 10(-3) + 1} = \sqrt{48} = 4\sqrt{3}$. **Ans.(2)**

45. The radical axis of the coaxial system of circles with limiting points (1, 2) and (4, 3) is given by the equation :
(1) $3x + y - 10 = 0$ (2) $3x - y + 10 = 0$ (3) $3x + y + 10 = 0$ (4) none of these

Sol. $3x + y - 10 = 0$. **Ans.(1)**

46. How many tangents can be drawn to a circle, parallel to a given line ?
(1) 1 (2) 2 (3) infinite (4) none of these

Sol. **Ans.(2)**

47. The equation of the parabola with focus (–8, –2) and directrix $y = 2x - 9$ is :
(1) $x^2 + y^2 + 2xy + 16x + 2y + 253 = 0$ (2) $x^2 + 4y^2 + 4xy + 16x + 2y + 259 = 0$
(3) $4x^2 + y^2 + 4xy + 16x + 2y + 257 = 0$ (4) none of these

Sol. Let P (x, y) be any point on the parabola
$\therefore$ Distance of P from focus (–8, –2) = Distance of P from directrix $2x - y - 9 = 0$.

$\therefore \sqrt{(x+8)^2 + (y+2)^2} = \frac{2x - y - 9}{\sqrt{2^2 + 1^2}}$ or, $5[(x + 8)^2 + (y + 2)^2] = (2x - y - 9)^2$ or,

$x^2 + 4y^2 + 4xy + 116x + 2y + 259 = 0$. Which is the required equation of parabola. **Ans.(2)**

48. If the latus rectum of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is equal to half of its minor axis, then the eccentricity of the ellipse is :

(1) $e=\frac{3}{2}$ (2) $\frac{\sqrt{3}}{2}$ (3) $\frac{2}{\sqrt{3}}$ (4) none of these

Sol. Latus rectum is $\frac{2b^2}{a}$. Given latus rectum = 1/2 (2b)

$\frac{2b^2}{a}=\frac{1}{2}\cdot(2b);\frac{b}{a}=\frac{1}{2};\ b=\frac{a}{2};e^2=\frac{a^2-b^2}{a^2}\frac{a^2-a^2/4}{a^2}=\frac{3}{4};\ e=\frac{\sqrt{3}}{2}$. **Ans.(2)**

49. Equation of ellipse when b > a is :

(1) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ (2) $\frac{y^2}{a^2}+\frac{x^2}{b^2}=1$ (3) $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ (4) $\frac{x^2}{a^2}-\frac{y^2}{b^2}=-1$

Sol. **Ans.(1)**

50. Conic represented by the equation $y^2 - 2xy\sqrt{3} + 3x^2 + 6x - 4y + 5 = 0$ is :
(1) parabola (2) ellipse (3) hyperbola (4) none of these.

Sol. Here $\Delta \neq 0$ and $h^2 = ab$. Hence the given equation represents a parabola. **Ans.(1)**

51. The locus of perpendicular tangents for the curve $\frac{x^2}{3}+\frac{y^2}{4}=1$ is :

(1) $x^2 + y^2 = 3$ (2) $x^2 + y^2 = 7$ (3) $x^2 + y^2 = 6$ (4) $x^2 + y^2 = 16$

Sol. The locus of perpendicular tangents is the director circle whose equation for ellipse is $x^2 + y^2 = a^2 + b^2$ i.e. $x^2 + y^2 = 3 + 4 = 7$. **Ans.(2)**

52. Equation of two circles were given, then the equation of circle with common chord as diameter was required sorry could not get data.

53. Distance of an arbitary point on the ellipse from the focus is 4 units and distance from directrix is 6 units, then eccentricity of ellipse is :

(1) $\frac{2}{3}$ (2) $\frac{3}{2}$ (3) $\sqrt{\frac{2}{3}}$ (4) none of these.

Sol. Eccentricity = $\frac{\text{distance from focus}}{\text{distance from directrix}}=\frac{4}{6}=\frac{2}{3}$. **Ans.(1)**

54. $\int_{-1}^{4}|x|dx = ?$

(1) $\frac{15}{2}$ (2) $\frac{17}{2}$ (3) 8 (4) none of these.

Sol. $\int_{-1}^{4}|x|dx = \int_{-1}^{0}-x\ dx + \int_{0}^{4}x\ dx = \left.\frac{-x^2}{2}\right|_{-1}^{0} + \left.\frac{x^2}{2}\right|_{0}^{4} = \frac{1}{2}+8=\frac{17}{2}$. **Ans.(2)**

55. The following is a monthly trend equation $y_c = 40 + 4x$ [Origin Jan. 1982, x unit = one month, y unit = month sales (in 000) rupees]. The annual trend equation for it will be :
(1) $y_c = 480 + 576x$ (2) $y_c = 480 + 4x$ (3) $y_c = 744 + 576x$ (4) $y_c = 748 + 848x$

Sol. Shifting origin to 1st July 1982, the new trend equation will by $y_c = 40 + (x + 5.5) = 62 + 4x$. Converting it into an annual trend equation, it takes the form
$y_c = (62 \times 12) + (4x \times 144) = 744 + 576x$. **Ans.(3)**

56. The marks obtained by nine students in physics and mathematics are given below :

Marks in physics	35	23	47	17	16	43	9	6	28
Marks in Mathematics	30	33	45	23	8	49	12	4	31

Then the Spearman's Rank correlation coefficient for the above data is :
(1) 0.8 (2) 0.9 (3) 0.75 (4) none of these.

Sol. Rank table for the given data is as follows :

x	Ranks in x R_1	y	Ranks in y R_2	$Di = R_1 - R_2$	D_i^2
35	3	30	5	– 2	4
23	5	33	3	2	4
47	1	45	2	– 1	1
17	6	23	6	0	0
16	7	8	8	– 1	1
43	2	49	1	1	1
9	8	12	7	1	1
6	9	4	9	0	0
28	4	31	4	0	0
					$\sum D_i^2 = 12$

$$\rho = \left[1 - \frac{6\,(\sum D_i^2)}{N\,(N^2 - 1)}\right] = \left[1 - \frac{6 \times 12}{9\,(81 - 1)}\right] = 0.9$$ **Ans.(2)**

57. If $r = \sqrt{b_{yx} \times b_{xy}}$, then range of r is :
(1) $-1 \le r \le 1$ (2) $r \le 0$ (3) $r < 0$ (4) $-3 \le r \le 3$

Sol. Standard result. **Ans.(1)**

58. Mode for the series 2, 9, 3, 4, 9, 6, 9, 2, 12 is :
(1) 2 (2) 9 (3) 2 (4) 3

Sol. **Ans.(2)**

59. The mean, median and standard deviation of 100 observations are found to be 90, 84 and 72 respectively, then Karl Pearson's coefficient of skewness is :
(1) 0.25 (2) 0.33 (3) 0.66 (4) 0.11

Sol. $j_p = \frac{3(\text{Mean} - \text{Median})}{\text{S.D.}} = \frac{3(90 - 84)}{72} = 0.25$. **Ans.(1)**

60. Equation of y on x is 2x + 3y – 10 = 0 and equation of x on y is 4x + y – 5 = 0, then b_{yx} is :

(1) $\frac{-3}{2}$ (2) $\frac{-2}{3}$ (3) $\frac{-1}{4}$ (4) – 4

Sol. b_{yx} = Slope of line of regression of y on x = $\frac{-2}{3}$. **Ans.(2)**

61. In a distribution sum of upper and lower quartiles is 28 and the median is 11 and difference of upper and lower quartiles is 12. The Bowley's coefficient of skewness is equal to :

(1) 0.3 (2) 0.4 (3) 0.5 (4) 0.45

Sol. $S_B = \frac{Q_3 + Q_1 - 2\text{ Md.}}{Q_3 - Q_1} = \frac{28 - 2(11)}{12} = \frac{6}{12} = 0.5$. **Ans.(3)**

62. The standard deviation for the following data is :

x	6	7	8	9	10	11	12
f	3	6	9	13	8	5	4

(1) 1.4 (2) 1.5 (3) 1.6 (4) 1.75

Sol.

x	f	fx	deviations from mean d	d^2	fd^2
6	3	18	– 3	9	27
7	6	42	– 2	4	24
8	9	72	– 1	1	9
9	13	117	0	0	0
10	8	80	1	1	8
11	5	55	2	4	20
12	4	48	3	9	36
Total	48	432			124

Mean = $\frac{432}{42} = 9$.

Standard deviation = $\sigma = \sqrt{\frac{\Sigma fd^2}{N}} = \sqrt{\frac{124}{48}} = 1.6$. **Ans.(3)**

63. The mean and standard deviation of a sample of 100 observation was 40 and 10 respectively, then the coefficient of variation is :

(1) 4 (2) 400 (3) 10.25 (4) 25

Sol. C.V. = $\frac{\sigma}{x} \times 100 = \frac{10}{40} \times 100 = 25$. **Ans.(4)**

64. The arithmetic mean for the following frequency distribution is :

Class	0 – 4	4 – 8	8 – 12	12 – 16	16 – 20
Frequency	6	11	20	7	4

(1) 8 (2) 9.2 (3) 9.33 (4) 9.66

Sol.

Class	M.d. value (x)	f_1	$u_i = \frac{x - A}{4} = \frac{x - 10}{4}$	$f_i u_i$
0 – 4	2	6	– 2	– 12
4 – 8	6	11	– 1	– 11
8 – 12	10	20	0	0
12 – 16	14	7	1	7
16 – 20	18	4	2	8
		$\sum f_i = 48$		$\sum f_i u_i = -8$

Thus $\bar{x} = 10 + 4\left(\frac{-8}{48}\right) = 9.33$. **Ans.(3)**

65. Sum of edges of a cube is 48. The volume of cube is :
(1) 64 (2) 216 (3) 512 (4) none of these

Sol. There are 12 edges in a cube.

Length of 1 edge = $\frac{48}{12} = 4$.

Volume of cube = $4^3 = 64$. **Ans.(1)**

66. There are four operations performed on 2 positive whole numbers. First the 2 were added, then the lesser was subtracted from the greater, then they were multiplied together and finally the larger number was divided by the smaller. The result thus obtained were then added to get 243. The numbers are :
(1) 54, 8 (2) 27, 3 (3) 51, 8 (4) 24, 8

Sol. Check from options. **Ans.(4)**

67. $\sqrt{67}$ is nearly equal to :
(1) 8.1 (2) 8.2 (3) 8.3 (4) 8.4

Sol. $\sqrt{67} \approx 8.2$. **Ans.(2)**

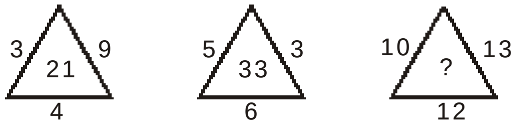
68. Next number in the series 759, 513, 267, ... is :
(1) 121 (2) 143 (3) 63 (4) none of these.

Sol. Series is in A.P. with common difference – 246. So next term = 21. **Ans.(4)**

69. Next number in the series 729, 19^2, 17^2, ... is :
(1) 25 (2) 9 (3) 49 (4) none of these.

Sol. Series is 27^2, 19^2, 17^2, 9^2, so the next term will be 7^2, i.e. 49. **Ans.(3)**

70. What will come in place of '?' ?



(1) 127 (2) 129 (3) 131 (4) 133

Sol. Inner element
(1) $3 \times 4 + 9 = 21$
(2) $6 \times 5 \times 3 = 33$
(3) $10 \times 12 + 13 = 133$. **Ans.(4)**

71. A cyclist rides with half the speed with which a motorcyclist rides. If motorcyclist rides with the speed of 50 km/hr. Then the distance covered by a cyclist in 6 minutes is :
(1) 150 m (2) 2.5 km (3) 5 km (4) 1.5 km

Sol. Speed of cyclist = $\frac{50}{2} = 25\text{ km/hr}$

In 60 minutes he covers 25 km.

$\therefore$ In 6 minutes he will cover $\frac{25}{10} = 2.5\text{ km}$. **Ans.(2)**

72. A clock strikes 1 time at 1'o clock, 2 times at 2'o clock and so on upto 24 hours. How many times the clock strikes in 24 hours ?
(1) 300 times (2) 156 times (3) 48 times (4) 96 times

Sol. Till 12'o clock no. of strikes = 1 + 2 + ... 12 = 78
After it, clock will again strike 1 time at 1'o clock 2 times at 2'o clock and so on upto 12'o clock.
Total no. of strikes = 2(78) = 156. **Ans.(2)**

73. $\frac{d}{dx}(\tan^{-1} e^x) =$

(1) $\frac{1}{1 + e^{2x}}$ (2) $\frac{e^x}{(1 + e^x)^2}$ (3) $\frac{e^x}{1 + e^{2x}}$ (4) none of these

Sol. $\frac{d}{dx}(\tan^{-1} e^x) = \frac{1}{1 + e^{2x}} \cdot e^x = \frac{e^x}{(1 + e^{2x})}$. **Ans.(3)**

74. If u = log (tan x + tan y), then $\sin 2x \frac{\partial u}{\partial x} + \sin^2 y \frac{\partial u}{\partial y}$ is equal to :

(1) 0 (2) 2 (3) 4 (4) 24

Sol. $\frac{\partial u}{\partial x} = \frac{\sec^2 x}{\tan x + \tan y} \cdot \frac{\partial u}{\partial y} = \frac{\sec^2 y}{(\tan x + \tan y)}$

$$\sin 2x \frac{\partial u}{\partial x} + \sin 2y \frac{\partial u}{\partial y} = \sin 2x \left(\frac{\sec^2 x}{\tan x + \tan y}\right) + \sin 2y \left(\frac{\sec^2 y}{\tan x + \tan y}\right)$$

$$= \frac{2 \tan x}{\tan x + \tan y} + \frac{2 \tan y}{\tan x + \tan y} = 2$$. **Ans.(2)**

75. The volume of a spherical balloon is increasing at the rate of 20 cm^3/s. The rate of change of its surface area at the instant when its radius is 8 cm is :
(1) 5 cm^2/s (2) 10 cm^2/s (3) 4 cm^2/s (4) 2.5 cm^2/s

Sol. Here $\frac{dV}{dt} = 20\ cm^3/s$

$V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{5}{\pi r^2}$

Now $S = 4\pi r^2 \Rightarrow \frac{dS}{dt} = 8\pi r \frac{dr}{dt} = 8\pi r\frac{5}{\pi r^2} = \frac{40}{r}$

$\frac{dS}{dt} = \left(\frac{40}{8}\right) = 5\ cm^2/sec.$ **Ans.(1)**

76. The largest value of $2x^3 - 3x^2 - 12x + 5$ for $-2 \le x \le 4$ occurs when x is equal to :
(1) −2 (2) −1 (3) 2 (4) 4

Sol. $\frac{dy}{dx} = 0 \Rightarrow 6x^2 - 6x - 12 = 0$. Max. at x = −1 Min. at x = 2 by sign of $\frac{d^2y}{dx^2} = 12x - 6$.
But f(−2) = −1, f(−1) = 3, f(2) = −15, f(4) = 37. Hence the largest value in the interval is 37 which occurs at x = 4. **Ans.(4)**

77. The point of inflexion for the curve $x^5 - 5x^4 + 5x^3 - 1$ is :
(1) 0 (2) 1 (3) 3 (4) does not exist

Sol. $f(x) = x^5 - 5x^4 + 5x^3 - 1$.
Differentiating $f'(x) = 5x^2(x - 3)(x - 1)$
critical points are x = 0, 1, 3.
$f''(x) = 20x^3 - 60x^2 + 30x$.
$\therefore$ f"(0) = 0. But f"(0) ≠ 0, so x = 0 is a point of inflexion. **Ans.(1)**

78. The function $f(x) = x^4 - 2x^2$ increases in the interval :
(1) $(-\infty, -1) \cup (0, 1)$ (2) $(-1, 1)$ (3) $(-1, 0) \cup (1, \infty)$ (4) R (−1, 1)

Sol. We have $f(x) = x^4 - 2x^2$. $\therefore f'(x) = 4x^3 - 4x = 4x(x^2 - 1)$.
For f(x) to be increasing : $f'(x) > 0 \Rightarrow 4x(x^2 - 1) > 0 \Rightarrow x(x^2 - 1) > 0$
$\Rightarrow (x > 0 \text{ and } x^2 - 1 > 0) \text{ or } (x < 0 \text{ and } x^2 - 1 < 0)$
$\Rightarrow x > 0 \text{ and } (x - 1)(x + 1) > 0 \text{ or } x < 0 \text{ and } (x - 1)(x + 1) < 0$
$\Rightarrow x > 0 \text{ and } (x < -1 \text{ or } x > 1) \text{ or } x < 0 \text{ and } (-1 < x < 1)$
$\Rightarrow x > 1 \text{ or } -1 < x < 0 \Rightarrow x \in (1, \infty) \cup (-1, 0)$.
So, f(x) is increasing on $(-1, 0) \cup (1, \infty)$.
For f(x) to be decreasing : $f'(x) < 0 \Rightarrow 4x(x^2 - 1) < 0 \Rightarrow x(x^2 - 1) < 0$
$\Rightarrow (x > 0 \text{ and } x^2 - 1 < 0) \text{ or } (x < 0 \text{ and } x^2 - 1 > 0) \Rightarrow x > 0$ and
$(x - 1)(x + 1) < 0$ or $x < 0$ and $(x - 1)(x + 1) > 0 \Rightarrow x > 0$ and $-1 < x < 1$ or $x < 0$ and ($x < -1$ or $x > 1$) [as $(x - 1)(x + 1) < 0 \Rightarrow -1 < x < 1$ and
$(x - 1)(x + 1) > 0 \Rightarrow x < -1$ or $x > 1$] $\Rightarrow 0 < x < 1$ or $x < -1$
$\Rightarrow x \in (0, 1) \cup (-\infty, -1)$. So, f(x) is decreasing on $(-\infty, -1) \cup (0, 1)$.
Aliter : $f'(x) = 4x^3 - 4x = 4x(x^2 - 1)$
For x = 0.5 f'(x) < 0, so x = 0.5 should not be the part of the answer. Moreover for x > 1 f'(x) > 0. The only option which satisfies both the conditions is (3). **Ans.(3)**

79. The length of tangent for a curve at point (2, 3) is 5. Then the length of normal at that point will be :

(1) 5 (2) $5\frac{1}{4}$ (3) 6 (4) $6\frac{1}{4}$

Sol. Length of tangent = $y\sqrt{1+\left(\frac{dy}{dx}\right)^2} = 5$ or $\left(\frac{dy}{dx}\right)^2 = \frac{25-y^2}{y^2} \Rightarrow \left(\frac{dy}{dx}\right)^2 = \frac{y^2}{25-y^2}$

Length of normal $= y\sqrt{1+\left(\frac{dy}{dx}\right)^2} = 5\sqrt{\frac{25}{25-y^2}} = \frac{25}{4} = 6\,\frac{1}{4}$. **Ans.(4)**

80. The equation of tangent for the curve $y = e^x$ at the point whose abscissa is 1 is :

(1) y = ex (2) y = ex + 2e (3) y = ex + 4e (4) ey = x + 3

Sol. $y = e^x \Rightarrow \frac{dy}{dx} = e^x$

Equation of tangent is (y – e) = e (x – 1), y – ex = 0. **Ans.(1)**

81. $\int_0^{\pi/2} \sin^3 x \, dx =$

(1) 0 (2) $\frac{2}{3}$ (3) $\frac{\pi}{3}$ (4) none of these

Sol. $\int_0^{\pi/2} \sin^3 x \, dx = \frac{2}{3}$ (By Walli's formula). **Ans.(2)**

82. Area of a loop of the curve $r^2 = a^2 \sin 2\theta$ is :

(1) a^2 (2) $\frac{a^2}{2}$ (3) $\frac{a^2}{4}$ (4) $\frac{a^2}{8}$

Sol. Putting r = 0, we get $\sin 2\theta = 0$ or $\theta = 0, \frac{\pi}{2}$.

∴ For the first loop θ varies from 0 to $\frac{\pi}{2}$.

∴ Required area of loop = $\frac{1}{2}\int_0^{\pi/2} r^2 \, d\theta = \frac{1}{2} a^2 \int_0^{\pi/2} \sin 2\theta = \frac{1}{2} a^2$. **Ans.(2)**

83. The figure bounded by the graphs of $y^2 = 4x$, y = 0 and x = 1 is rotated around the line x = 1. The volume of the resulting solid is :

(1) $\frac{16\pi}{5}$ (2) $\frac{15\pi}{16}$ (3) $\frac{5\pi}{16}$ (4) $\frac{16\pi}{5}$

Sol. The curve bounded by $y^2 = 4x$ and x = 1 revolved about the line AC (x = 1).

∴ Required volume $V = \int_0^2 \pi\,(1-x)^2 \, dy = \int_0^2 \pi\left(1-\frac{y^2}{4}\right)^2 dy = \frac{16\pi}{15}$. **Ans.(1)**

84. If I_1, I_2 are integrating factors of the equations $xy' + 2y = 1$ and $xy' - 2y = 1$ then :

(1) $I_1 = -I_2$ (2) $I_1I_2 = x^2$ (3) $I_1 = x^2I_2$ (4) $I_1I_2 = 1$

Sol. The given equations are $xy' + 2y = 1$ ...(i) and $xy' - 2y = 1$ ...(ii).
Both differential equations are linear, Therefore I.F of (i),

$I_1 = e^{\int \frac{2}{x}dx} = e^{2\log x} = x^2$ ...(iii) and I.F. of (ii), $I_2 = e^{-\int \frac{2}{x}dx} = e^{-2\log x} = \frac{1}{x^2}$ ...(iv).

Therefore from (iii) and (iv) $I_1I_2 = 1$. **Ans.(4)**

85. Solution of the differential equation $\frac{dy}{dx} = 1 + x + y + xy$ is :

(1) $e^{(1-x)^2/2}$ (2) $e^{(1-x)^2/2} - 1$ (3) $\log_e(1 + x) - 1$ (4) none of these

Sol. $\frac{dy}{dx} = (1+x)(1+y) \Rightarrow \frac{dy}{(1+y)} = (1+x)\ dx$

$\Rightarrow \log(1 + y) = x + \frac{x^2}{2} + C$. **Ans.(4)**

86. The solution curve of the equation $y = 2 - \left(\frac{y}{x}\right)$ which passes through the point (1, 2) is given by :

(1) $y = x + \frac{1}{x}$ (2) $y = x - \frac{1}{x}$ (3) $y^2 = x + \frac{1}{x} + 2$ (4) $y = x^2 - x + 2$

Sol. The given differential equation is $\frac{dy}{dx} = 2 - \frac{y}{x}$ or $\frac{dy}{dx} + \frac{y}{x} = 2$ ...(i). Therefore $I.F = e^{\int \frac{1}{x}dx} = e^{\log x} = x$.
The solution of (i) is $y \cdot x = \int 2 \cdot x\ dx + c$ or $xy = x^2 + c$ ...(ii). Since (ii) passes through (1, 2).
Therefore $2 = 1 + c \Rightarrow c = 1$. Therefore the required solution curve is

$xy = x^2 + 1$ or $y = x + \left(\frac{1}{x}\right)$. **Ans.(1)**

87. Solution of the differential equation $\frac{dy}{dx} = \cos(x + y)$ is :

(1) $\frac{1}{2}\sec^2(x + y) = x + C$ (2) $\cos\left(\frac{x+y}{2}\right) = x + C$

(3) $\sin\left(\frac{x+y}{2}\right) = x + C$ (4) $\tan\left(\frac{x+y}{2}\right) = x + C$

Sol. Put $x + y = v$, so that $\frac{dy}{dx} = \frac{dv}{dx} - 1$.

So, the given equation becomes $\frac{dv}{dx} - 1 = \cos v$

$\frac{dv}{1 + \cos v} = dx \Rightarrow \frac{1}{2}\sec^2\frac{v}{2}\ dv = dx$

Integrating $\tan\frac{v}{2} = x + C$ or $\tan\left(\frac{x+y}{2}\right) = x + C$. **Ans.(4)**

88. Suppose for every integer n, $\int_{n}^{n+1} f(x)\,dx = n^2$ the value of $\int_{-2}^{4} f(x)\,dx$ is :

(1) 16 (2) 14 (3) 19 (4) none of these

Sol. Given $\int_{n}^{n+1} f(x)\,dx = n^2$

$\therefore \int_{-2}^{4} f(x)\,dx = \int_{-2}^{-1} f(x)\,dx + \int_{-1}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx + \int_{1}^{2} f(x)\,dx + \int_{2}^{3} f(x)\,dx + \int_{3}^{4} f(x)\,dx$

$= (-2)^2 + (-1)^2 + (0)^2 + (1)^2 + (2)^2 + (3)^2 = 4 + 1 + 0 + 1 + 4 + 9 = 19$. **Ans.(3)**

89. Solution of the differential equation $dy = \frac{y^2}{x}dx$ is :

(1) $e^{-y} = cx$ (2) $e^{-1/y} = cx$ (3) $e^{1/y} = cx$ (4) $e^{2/y} = cx$

Sol. $dy = \frac{y^2}{x}dx$ or $\frac{dy}{y^2} = \frac{dx}{x}$

Integrating $\frac{-1}{y} = \log x + \log c$

$\frac{-1}{y} = \log cx$ or $e^{-1/y} = cx$. **Ans.(2)**

90. The population of a city doubles in 50 years. Assuming that rate of increase is proportional to the number of inhabitants. In how many years will it treble itself ?

(1) 82 (2) 79 (3) 75 (4) 70

Sol. $\frac{dP}{dt} \propto P \Rightarrow \frac{dP}{dt} = kP$

Integrating $\log P = kt + \log P_0$, where P_0 is the initial population

$\Rightarrow \log\frac{P}{P_0} = kt$ ...(1)

or $\log 2 = 50\,k \Rightarrow k = \frac{1}{50}\log 2$

Again from (1) $\log\frac{3P_0}{P_0} = kt = \frac{1}{50}\log 2.t \Rightarrow t = 50\frac{\log 3}{\log 2} \approx 79$ years . **Ans.(2)**

91. $\int 5^{5^{5^x}}.5^{5^x}.5^x dx$ is equal to :

(1) $\frac{5^{5^x}}{(\log 5)^3} + C$ (2) $5^{5^{5^x}}(\log 5)^3 + C$ (3) $\frac{5^{5^{5^x}}}{(\log 5)^3} + C$ (4) none of these

Sol. Put $5^{5^{5^x}} = t$, $5^{5^{5^x}}.5^{5^x}.5^x.(\log 5)^3 dx = dt \frac{1}{(\log 5)^3}\int 1\,dt = \frac{1}{(\log 5)^3} = \frac{5^{5^{5^x}}}{(\log 5)^3} + C$. **Ans.(3)**.

92. A river is 80 feet wide. The depth d (in feet) of the river at a distance of x feet from one bank is given by the following table :

x	0	10	20	30	40	50	60	70	80
d	0	4	7	9	12	15	14	8	3

By Simpson's rule, the area of the cross–section of the river is :
(1) 705 sq. feet (2) 690 sq. feet (3) 710 sq. feet (4) 715 sq. feet

Sol. $\int_0^{80} y\,dx = \frac{10}{3}[0 + 3 + 4(4 + 9 + 15 + 8) + 2(7 + 12 + 14)] = \frac{10}{3}[3 + 4(36 + 2 \times 33)]$

$= \frac{10}{3}[3 + 144 + 66] = \frac{10}{3}[144 + 69] = \frac{10}{3} \times 216 = 710$ sq. feet. **Ans.(3).**

93. The solution of the difference equation $y_{n+2} + y_{n+1} - 6y_n = 0$ is :
(1) $A(-2)^n + B(-3)^n$ (2) $y = A.2^n + B(-3)^n$ (3) $A(-2)^n + B(3)^n$ (4) none of these

Sol. Auxiliary equation for the given difference equation is $(m^2 + m - 6) = 0 \Rightarrow m = 2$ or -3.
General solution for the equation is $y_n = A.(2)^n + B(-3)^n$. **Ans.(2)**

94. Solution of the difference equation $y_{k+1} - 2y_k = k^2$ is :
(1) $A(2)^k - (k^2 + 2k + 2)$ (2) $A(2)^k - 2k$
(3) $A(2)^k - (k^2 + 2)$ (4) none of these

Sol. General solution is $y_k = A(2)^k$

Particular solution $= \frac{1}{E-2}k^2 = -\frac{1}{1-\Delta}\left[k^{(2)} + k^{(1)}\right] = -(k^2 + 2k + 3)$

Complete solution is $y_k = A(2)^k - (k^2 + 2k + 3)$. **Ans.(4)**

95. In a survey of 200 students of a higher secondary school, it was found that 120 studied mathematics; 90 studied physics; and 70 studied chemistry; 40 studied mathematics and physics; 30 studied physics and chemistry; 50 studied chemistry and mathematics, and 20 studied none of these subjects. The number of students who studied all the three subjects is :
(1) 10 (2) 8 (3) 20 (4) 15

Sol. We have $200 - 20 = 120 + 90 + 70 - 40 - 30 - 50 + x \Rightarrow x = 20$. **Ans.(3)**

96. With the help of trapezoidal rule for numerical integration and the following table :

x	0	0.25	0.50	0.75	1
f(x)	0	0.0625	0.2500	0.5625	1

The value of $\int_0^1 f(x)\,dx$ is :

(1) 0.35342 (2) 0.34375 (3) 0.34457 (4) 0.33334

Sol. $\int_0^1 f(x)\,dx = \frac{0.25}{2}[(0 + 1) + 2(0.0625 + 0.2500 + 0.5625)] = 0.34375$. **Ans.(2)**

97. If 4 * 3 = 28

6 * 8 = 108

then 10 * 12 is equal to :

(1) 144   (2) 286   (3) 256   (4) 168

Sol. Here $a * b = a^2 + b^2 + b$

Since $4 * 3 = 4^2 + 3^2 + 3 = 28$

$6 * 8 = 6^2 + 8^2 + 8 = 108$

$\therefore$ $10 * 12 = 10^2 + 12^2 + 12 = 256$. **Ans.(3)**

98. The smallest five digit number that can be formed using the digits 0, 2, 4, 7, 9 is :

(1) 02479   (2) 20749   (3) 20794   (4) none of these

Sol. The smallest five digit number than can be formed is 20479. **Ans.(4)**

99. Find the two digit number given that its unit's digit exceeds the ten's digit by 2 and that the product of the desired number by the sum of its digits is equal to 144.

(1) 42   (2) 24   (3) 35   (4) 64

Sol. Check from options. **Ans.(2)**

100. Ram takes money from the employees cooperative society at lower rate of interest and saves in a scheme, which gives him a compound interest of 20% compounded annually. The least number of complete years after which his sum will be more than doubled is :

(1) 2   (2) 4   (3) 6   (4) 8

Sol. $P\left(1+\frac{20}{100}\right)^T > 2P \Rightarrow (1\cdot 2)^T > 2 \Rightarrow T = 4$. **Ans.(2)**